Discrete-Time Analysis of an ATM Multiplexer

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Technical Report 1704
September 1995

Naval Command, Control and
Ocean Surveillance Center
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92152-5001

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ADMINISTRATIVE INFORMATION

The work detailed in this report was performed for the Office of Naval Research by the Naval Command, Control and Ocean Surveillance Center, RDT&E Division, Networking Design and Analysis Branch, Code 827. Funding was provided under Program Element 0601152N, Accession Number DN304006.

Released by
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Analysis Branch

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EXECUTIVE SUMMARY

OBJECTIVE

This report proposes a discrete-time model of an Asynchronous Transfer Mode (ATM) statistical multiplexer subject to heterogeneous groups of on-off traffic sources with correlated burst periods. NRaD engineers have formulated algorithms to compute both the exact transient and steady-state cell-loss probabilities of the multiplexer.

METHOD

The complexity of the algorithms is exponential to the number of traffic types multiplexed; therefore, it may be impractical to use them to address realistic ATM design problems. For this reason, NRaD engineers formulated and analyzed an approximation model of the multiplexer subject on-off traffic sources with random burst periods. They developed an algorithm to calculate the exact cell-loss probability of the approximation multiplexer model. Unfortunately, the time complexity of the algorithm is also exponential to the number of traffic types multiplexed. To circumvent this difficulty, the engineers formulated an efficient approximation procedure to estimate the cell-loss probability of the approximation multiplexer model.

CONCLUSION

NRaD engineers developed algorithms to compute both multiplexer models' cell-loss probabilities. The algorithms' results have been compared to simulation outputs and they have showed perfect agreement (within noise level). The algorithms will be invaluable for resolving basic ATM design issues such as buffer dimensioning, maximal loading, and admission control.

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1. BACKGROUND

Asynchronous Transfer Mode (ATM) networks will dominate future global telecommunication industries. These networks will make many telecommunication services (e.g., video conferencing and high-speed data) economical and, in turn, will bring fundamental changes to the way that people communicate and travel. Despite the expected prominence of ATM, many basic ATM design issues, such as buffer dimensioning and routing, still have not been addressed. The resolution of these issues will require a thorough understanding of the dynamic behavior of the ATM multiplexer. The analysis of the cell loss behavior of an ATM multiplexer is the focus of this report.

2. MATHEMATICAL MODEL OF THE ATM MULTIPLEXER

One classical approach to understanding a system's behavior is to design a mathematical model that captures the salient characteristics of the system and then analyze the model. This approach will be used in this report.

Statistical multiplexing is a means of sharing a communication channel by many traffic sources simultaneously. A statistical multiplexer consists of a channel, a buffer, and a set of traffic sources sharing the channel. Traffic is generated by the sources and removed by the channel. Whenever the instantaneous aggregate traffic rate of the traffic sources is below the channel capacity, the amount of the traffic in the buffer does not accumulate; however, when the total-traffic rate exceeds the capacity of the channel, the excess traffic will be stored in the buffer; and when the buffer is full, excess traffic will be lost.

The most difficult task in developing an accurate ATM statistical multiplexer model is the characterization of a traffic source. As one may expect, the traffic generation behavior of a traffic source is stochastic and differs depending on the traffic type. The problem is to develop a *general* traffic model that may be used to describe an *arbitrary* traffic source. The graph (figure 1) depicting the traffic generation rate of a traffic source as a function of time yields insight for the formulation of a satisfactory traffic-source model. Common to all bursty traffic sources, there are *burst* periods during which a traffic source would generate traffic close to some peak rate, P, and there are *idle* periods during which the source would generate close to nothing. While it is impossible to completely describe all the particular characteristics of a traffic source, there are three *fundamental* characteristics that are considered to be the most important in determining the cell loss behavior at a statistical multiplexer: the peak traffic rate, the mean traffic rate, and the expected burst period. It is imperative that a traffic source model would account for these three characteristics.

An arbitrary traffic source that has a peak traffic rate, P, a mean traffic rate, m, and an expected burst period, $E(BP)$, may be approximated by an on-off traffic source model that is described as follows:

An on-off traffic source of the i-th type alternates between on and off periods. The duration of an on period, measured in terms of integral multiple of a time slot of duration τ, is geometrically distributed with a parameter of α_i; the duration of an off period is geometrically distributed with a parameter of β_i, where $0 < \alpha_i, \beta_i < 1$. During its on periods, the source generates P_i cells per time slot and during its off periods, generates nothing. A schematic of a traffic source of the i-th type is shown in figure 2.

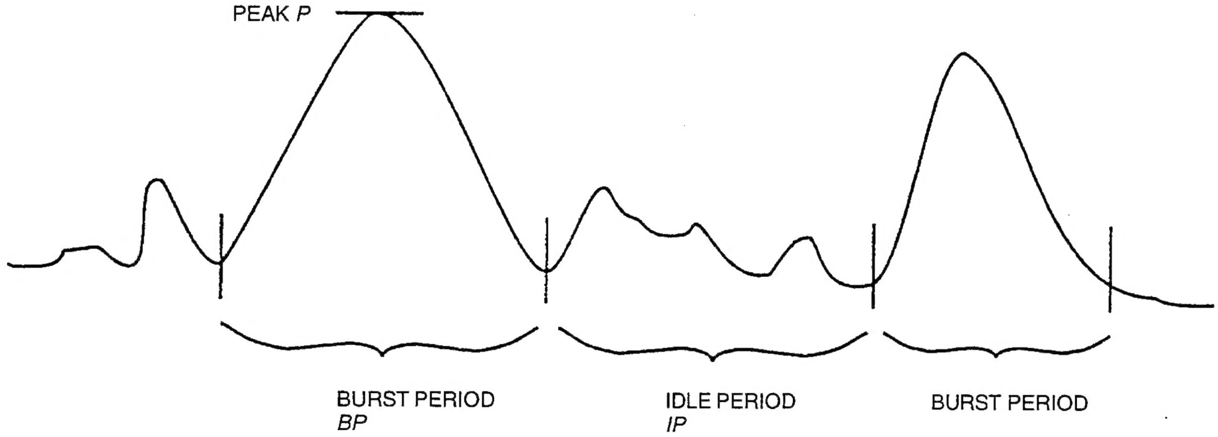

Figure 1. Traffic generation rate of a traffic source.

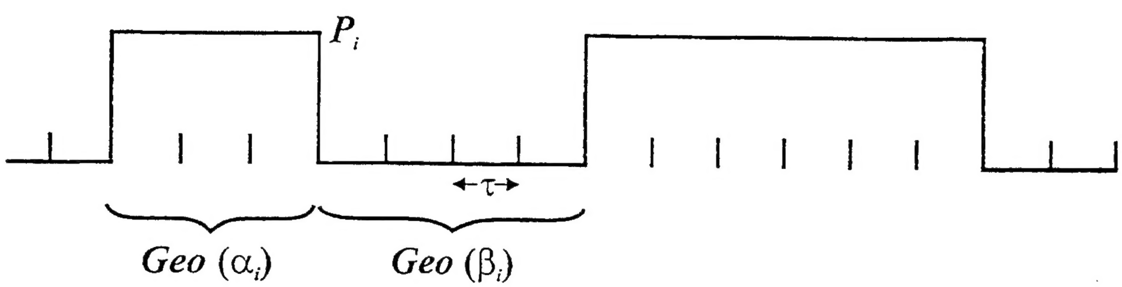

Figure 2. Traffic generation rate of an i-th type on-off source.

Notice that, given an arbitrary traffic source with peak rate, P, mean rate, m, and expected burst period $E(BP)$, one can construct an on-off traffic source approximation that has the same three fundamental characteristics. The parameters P_i, α_i, and β_i of the approximating on-off source are uniquely determined according to the following formulae:

- $P_i = P$
- $\alpha_i = 1 - \frac{1}{E(BP)}$
- $\beta_i = 1 - \frac{1}{E(BP)(\frac{P}{M} - 1)}$

Because of its extreme nature (generating at peak rate during its on period and nothing during its off periods), an on-off traffic source with fundamental characteristics P, m, and $E(BP)$ is likely to be more bursty than most traffic sources with the same characteristics; therefore, it would effect a higher cell loss at a multiplexer. This is particularly desirable for modeling purposes.

3. TRANSIENT CELL-LOSS PROBABILITY

A traffic type typically has two requirements: delay and loss. The delay requirement of a traffic type is expected to be met by an ATM network. The loss requirement is a bigger concern for the network. For this reason, the loss characteristics are the most important performance measure of an ATM multiplexer. The focus of this work is to formulate an efficient procedure that computes the *transient*, as well as *steady-state*, cell-loss probability of the multiplexer. A transient solution is desirable since it is expected that the behavior of an ATM network is highly dynamic and a steady-state solution alone may not provide adequate information for the control and design of an ATM network.

4. LITERATURE REVIEW

Statistical multiplexing, an efficient and flexible means of sharing a communication channel by many traffic sources simultaneously, is so critical to an ATM network that the term asynchronous transfer mode is just an alias for statistical multiplexing. Because a thorough understanding of the effect of statistically multiplexing diverse traffic sources onto a channel is required to address many basic ATM design problems, numerous attempts were made to analyze the multiplexer's behavior. The continuous-time analog of the on-off model developed in this report was first proposed in Anick, Mitra, and Sondhi (1982). The main motivation of that pioneering work was to evaluate the steady-state cell loss behavior of a statistical multiplexer subject to only one type of traffic stream. A closed-form expression for the steady-state distribution of the buffer content was derived assuming that the buffer size of the multiplexer is infinite. Although the analysis is elegant, it cannot be used in the control and design of an ATM network since only one traffic type was allowed in that model, whereas in a realistic ATM network, heterogeneous traffic types are expected.

In Kosten (1984), the on-off traffic model is extended to include heterogeneous types of on-off traffic sources. An approximation expression to estimate the steady-state probability of the buffer content exceeding a very large value is derived under the assumption that the buffer size is infinite. The validity of his approximation had never been extensively verified by simulation studies.

Kobayashi and Ren (1992) investigated a multiplexer subject to the same assumptions in Kosten (1984), but they focused on obtaining the transient probability of the buffer content exceeding a certain value. Unfortunately, they gave only a formulation, rather than an explicit solution of the problem. Shum (1994) derived an approximation expression for transient probability of the buffer content exceeding a large value; however, the accuracy of his expression is not uniformly satisfactory. To obtain an exact solution of the distribution of buffer content exceeding a certain value for the multiplexer model with an infinite size buffer requires one to solve a set of partial differential equations with very specialized boundary conditions; unfortunately, not even a satisfactory *numerical* procedure exists to solve these equations. A model with a *finite* buffer is even less tractable.

Currently, a satisfactory procedure that estimates the cell loss rate effected by heterogeneous on-off traffic sources at a finite-buffer statistical multiplexer does not exist, and developing such a procedure is the main motivation for this report.

5. ANALYSIS

We define the following variables:

- τ the duration of a time slot
- M the number of different traffic types
- N_i the number of traffic sources of the i-th type i=1, ..., M
- R_i the number of cells generated by an i-th type traffic source in a time slot during the source's on periods
- B the maximum number of cells that may be buffered at the multiplexer
- C the number of cells that may be transmitted by the output link of the multiplexer per time slot; We assume $1 \leq C$

We make the following simplifying assumptions:

a. An *on* period of a traffic source the i-th type lasts X_i time slots, where $Pr(X_i = j) = \alpha_i^{j-1}(1 - \alpha_i)$ and $1 \leq j$ and $0 < \alpha_i < 1$

b. An *off* period of a traffic source the i-th type lasts Y_i time slots, where $Pr(Y_i = j) = \beta_i^{j-1}(1 - \beta_i)$ and $1 \leq j$ and $0 < \beta_i < 1$

c. The number of cells at the buffer at a time k+1 is computed as $\max(\min(i + \underline{R} \cdot \underline{J} - C, B),0)$, where i is the number of cells in the buffer at time k; $\underline{J}$ is the state of the traffic sources at the start of time k

d. (Stationary Condition) $\sum_{i=1}^{M} R_i N_i \dfrac{\frac{1}{1-\alpha_i}}{\frac{1}{1-\alpha_i} + \frac{1}{1-\beta_i}} < C$

e. $\sum_{i=1}^{M} N_i R_i > C$

Assumptions a. and b. state that the on and off periods of an i-th type traffic source are geometrically distributed with parameters α_i and β_i, respectively. Assumption c. states that the number of cells at the beginning of the (k+1)-th time slot is equal to i, the number of cells at the buffer at the beginning of the k-th time slot, plus the number of cells generated during the k-th slot, $\underline{R} \cdot \underline{J}$, subtract the number of cells removed by the channel during the time slot, C; however, the number of cells in the buffer will never exceed the buffer size B and it will never drop below 0. Assumption d. states that the overall mean aggregate traffic rate of all sources is less than the capacity; this allows the number of cells at the multiplexer to have a steady-state distribution. Assumption e. ensures that we do not have a trivial problem, as $\sum_{i=1}^{M} R_i N_i \leq C$ implies that cell loss would never occur.

We denote:

- L_k the number of cells in the buffer at the beginning of the k-th time slot
- $L = \lim_{k \to \infty} L_k$ the number of cells in the buffer at the beginning of a time slot
- J_{ik} the number of the i-th type traffic sources that are on at the beginning of k-th time slot

- $S(M,\underline{N}) = \{(j_1,j_2,...,j_M) \mid 0 \le j_i \le N_i\, i = 1,2,...,M\}$
- $S(B,M,\underline{N}) = \{(i,\underline{j}) \mid 0 \le i \le B, \underline{j} \in S(M,\underline{N})\}$
- $\underline{J}_k = (J_{1k}, J_{2k}, ..., J_{Mk}) \in S(M,\underline{N})$
- $Pr(\underline{J}_{k+1} = \underline{l}_{k+1} \mid \underline{J}_k = \underline{l}_k)\ 0 \le k$, for $\underline{l}_{k+1}$ and $\underline{l}_k \in S(M,\underline{N})$ the probability that $\underline{J}_{k+1} = \underline{l}_{k+1}$, given that $\underline{J}_k = \underline{l}_k$
- $(L_k = j, \underline{J}_k = \underline{l})$ the joint event of $L_k = j$ and $\underline{J}_k = \underline{l}$.
- $Pr(L_{k+1} = j, \underline{J}_{k+1} = \underline{l} \mid L_k = i, \underline{J}_k = \underline{m}) \equiv P^k(j,\underline{l} \mid i,\underline{m})$ the probability that the state at time k+1 is $(j,\underline{l})$, given that the state at time k is $(i,\underline{m})$

The *transition probability* from the *state* $(L_k = i, \underline{J}_k = \underline{m})$ at time k to the state $(L_{k+1} = j, \underline{J}_{k+1} = \underline{l})$ at time k+1 is

$$P^k(j,\underline{l} \mid i,\underline{m}) = \xi\left\{ j = \max(\min(i + \sum_{x=1}^{M} R_x m_x - C, B), 0)\right\} \Pr\{\underline{J}_{k+1} = \underline{l} \mid \underline{J}_k = \underline{m}\}, \tag{1}$$

where $\xi(A)$ is the indicator function of the event A, i.e.,

$$\xi\{A\} = \begin{cases} 1 & A \text{ is true} \\ 0 & A \text{ is not true} \end{cases}$$

The transition probability for the state of the traffic sources from time k to k+1, $\Pr\{\underline{J}_{k+1} = \underline{l} \mid \underline{J}_k = \underline{m}\}$, is derived as follows:

For now, we only consider the change in the number of the on sources of the h-th type. Suppose at time k, the number of on sources of the h-th type is i. Let $\sigma_{h,k}$ denote the state of a traffic source of the h-type at time k; that is, $\sigma_{h,k}$ is either *on* or *off*. We conclude that

$$Pr(\sigma_{h,k+1} = off \mid \sigma_{h,k} = on) = 1 - \alpha_h,$$

$$Pr(\sigma_{h,k+1} = on \mid \sigma_{h,k} = on) = \alpha_h,$$

$$Pr(\sigma_{h,k+1} = on \mid \sigma_{h,k} = off) = 1 - \beta_h,$$

$$Pr(\sigma_{h,k+1} = off \mid \sigma_{h,k} = off) = \beta_h.$$

Let m be the number of the i on sources that stay on at time k+1; hence, $m \le i$ and $i - m$ sources turn off at time k+1. Let l be the number of the $N_h - i$ off sources that turn on at time k+1, where $l \le N_h - i$. We conclude that $N_h - i - l$ of the on sources at time k stay off at time k+1. Since the traffic sources turn on and off independently, we obtain

$$Pr(J_{h,k+1} = j \mid J_{h,k} = i) = \sum_{m=1}^{i} \sum_{\{l:l=m-j\}} \binom{i}{m}(\alpha_h)^m(1-\alpha_h)^{i-m}\binom{N-i}{l}(\beta_h)^{N_h-i-l}(1-\beta_h)^l, \tag{2}$$

where $\binom{x}{y} = \frac{x!}{y!(y-x)!}$ if $y \le x$ and $\binom{x}{y} = 0$ otherwise.

When N is large, it is preferable to use *Stirling's formula* to approximate $N!$ by $N^N e^{-N}\sqrt{2\pi N}$.

We conclude that since the traffic sources turn on and off independent of each other,

$$Pr(\underline{J}_{k+1} = \underline{l} \mid \underline{J}_k = \underline{m}) = \prod_{i=1}^{M} \Pr(J_{i,k+1} = l_i \mid J_{i,k} = m_i). \quad (3)$$

If the 2-tuple $(i,\underline{j}) \in S(B,M,\underline{N})$ were used to denote as a *state* of the multiplexer, then the set of states $S(B,M,\underline{N})$ forms a Markov chain. The transition probability from state $(i,\underline{m})$ at time k to state $(j,\underline{l})$ at time k+1 for the Markov chain is

$$Pr(L_{k+1} = j, \underline{J}_{k+1} = \underline{l} \mid L_k = i, \underline{J}_k = \underline{m}) = \xi\{[j = \max(\min(i + \underline{R} \cdot \underline{m} - C, B), 0)]\} \Pr\{\underline{J}_{k+1} = \underline{l} \mid \underline{J}_k = \underline{m}\}. \quad (4)$$

The *state evolution* equation of the multiplexer is

$$Pr(L_{k+1} = j, \underline{J}_{k+1} = \underline{l}) = \sum_{(i,\underline{m}) \in S(B,M,\underline{N})} Pr(L_{k+1} = j, \underline{J}_{k+1} = \underline{l} \mid L_k = i, \underline{J}_k = \underline{m}) Pr(L_k = i, \underline{J}_k = \underline{m}), \quad (5)$$

where L and $\underline{J}$ denote the number of cells in the buffer and the state of the traffic sources in steady state, respectively.

Given that the multiplexer is at state $(i,\underline{m}) \in S(B,M,\underline{N})$ at time 0, the probability that the state of the multiplexer at time k is $(j,\underline{l})$, that is, the transient probability that the multiplexer is at $(j,\underline{l})$, may be *recursively* determined according to (4), if one sets $Pr(L_0 = i, \underline{J}_0 = m) = 1$ and $Pr(L_0 = x, \underline{J}_0 = \underline{y}) = 0$ for $(x,\underline{y}) \neq (i,\underline{m})$. The time complexity of obtaining $Pr(L_k = j, \underline{J}_k = \underline{l})$ is $O(k(B\prod_{i=1}^{M} N_i)^2)$.

The probability that the state of the multiplexer is at $(j,\underline{l})$ at a *random epoch* sufficiently far away from time 0, that is, the steady-state probability that the multiplexer is at $(j,\underline{l})$, satisfies the following fixed-point equation:

$$Pr(L = j, \underline{J} = \underline{l}) = \sum_{(i,\underline{m}) \in S(B,M,\underline{N})} Pr(L = j, \underline{J} = \underline{l} \mid L = i, \underline{J} = \underline{m}) Pr(L = i, \underline{J} = \underline{m}), \quad (6)$$

where L and $\underline{J}$ denote the number of cells in the buffer and the state of the traffic sources in steady state, respectively. Although the probability $Pr(L = j, \underline{J} = \underline{l})$ may be computed recursively using equation (4) by letting $k \to \infty$, doing so would take a prohibitively large number of iterations. The steady-state probabilities may be obtained via the following iterative procedure:

$k = 0;$

for$((i,\underline{j}) \in S(B,M,\underline{N}))$

{

$(p^k)_{(i,\underline{j})} = \dfrac{1}{|S(B,M,\underline{N})|};$

};

for$((i,\underline{j}) \in S(B,M,\underline{N}))$

{

$(p^{k+1})_{(i,\underline{j})} = 0;$

};

While $\left(\left| p^{k+1}{}_{(i,\underline{j})} - p^{k}{}_{(i,\underline{j})} \right| \geq \delta\right.$ for each $(i,\underline{j}) \in S(B,M,\underline{N}))$

{

$k = k+1$;

for $((i,\underline{j}) \in S(B,M,\underline{N}))$

{

pick any ε_k such that $0 < \varepsilon_k < 1$;

$$p^{k}{}_{(i,\underline{j})} = (1 - \varepsilon_k) p^{k-1}{}_{(i,\underline{j})} + \varepsilon_k \sum_{(\ell,\underline{m}) \in S(B,M,\underline{N})} \xi\left\{ i = max(min(\ell + \underline{R}\cdot\underline{m} - C, B)0) \right\} \prod_{i=1}^{M} Pr(J_i = j_i \mid J_i = m_i) p^{k-1}{}_{(\ell,\underline{m})};$$

};

$$S = \sum_{(i,\underline{j}) \in S(B,M,\underline{N})} p^{k}{}_{(i,\underline{j})};$$

for $((i,\underline{j}) \in S(B,M,\underline{N}))$

{

$$p^{k}{}_{(i,\underline{j})} = \frac{p^{k}{}_{(i,\underline{j})}}{S};$$

};

};

The procedure terminates whenever it has found a consistent set of $p_{(i,\underline{j})}$'s that satisfy equation (6).

The condition $\sum_{i=1}^{M} R_i N_i \frac{\frac{1}{1-\alpha_i}}{\frac{1}{1-\alpha_i} + \frac{1}{1-\beta_i}} < C$ ensures that the Markov chain is ergodic. The ergodicity in turn ensures that a unique steady-state distribution solution exists. This implies that, if the procedure converges, it would converge to a unique solution. In general, existence of a solution does not imply convergence. Our computational experiences suggest that the procedure always converges, however.

The *space* complexity of the procedure is $O(B\prod_{i=1}^{M} N_i)$; the *time* complexity is $O(B^2\prod_{i=1}^{M} N_i^2)$.

Once that the state probability distribution of the multiplexer is determined, the cell loss rate at the multiplexer may be found as follows:

We define the following variables:

- $NL(L = i, \underline{J} = \underline{l})$ the number of cells that are lost at the end of a time slot, given that, at the start of the time slot, the number of cells in the buffer is i and the state of the traffic sources is $\underline{l}$
- $NA(L = i, \underline{J} = \underline{l})$ the number of cells that arrive during a time slot k, given that, at the start of the time slot, the number of cells in the buffer is i and the state of the traffic sources is $\underline{l}$
- $LR(L = i, \underline{J} = \underline{l})$ the cell loss rate during a time slot, given that the number of cells in the buffer is i at the start of k and the state of the traffic sources is $\underline{l}$

Note the following:

$$NL(L = i, \underline{J} = \underline{l}) = \max\left(\sum_{j=1}^{M} R_j l_j - (B - i + C), 0\right) \tag{7}$$

$$NA(L = i, \underline{J} = \underline{l}) = \sum_{j=1}^{M} R_j l_j \tag{8}$$

$$LR(L = i, \underline{J} = \underline{l}) = \frac{NL(L = i, \underline{J} = \underline{l})}{NA(L = i, \underline{J} = \underline{l})} \tag{9}$$

The cell-loss rate during a time slot is:

$$\sum_{(i,\underline{j}) \in S(B,M,\underline{N})} LR(L = i, \underline{J} = \underline{j}) \Pr(L = i, \underline{J} = \underline{j}) . \tag{10}$$

Equations (7) to (10) outline the steps to compute the steady-state cell-loss probability of the multiplexer once the steady-state probability of the multiplexer at each state $(i, \underline{m}) \in S(B, M, \underline{N})$ is found. Equations analogous to (7) to (10) may be developed to compute the transient cell-loss probability of the multiplexer once the transient-state probability of the multiplexer at each state $(i, \underline{m}) \in S(B, M, \underline{N})$ is known.

6. NUMERICAL EXAMPLES

We conducted simulation studies for two arbitrarily selected ATM multiplexing scenarios to evaluate the steady-state cell-loss probabilities projected by the algorithm (see tables 1 and 2). The results from the simulation studies and the algorithm are as follows:

6.1 EXAMPLE #1

$B = 2$
$C = 40$
$M = 2$
$N_1 = 2\ \alpha_1 = 0.2\ \beta 1 = 0.3\ R_1 = 20$
$N_2 = 1\ \alpha_2 = 0.8\ \beta 2 = 0.7\ R_2 = 10$

We denote $P_{(i,\underline{j})}$ as the steady-state probability that the buffer content of the multiplexer at the start of a time slot is i and that the state of the traffic sources is $\underline{j}$ and ϵ as the steady-state cell-loss probability. Relative error is defined as $100 \times \frac{\textit{simulation result} - \textit{analysis result}}{\textit{simulation result}}$.

Table 1. ATM multiplexing algorithm example #1.

	Analysis	Simulation	Relative Error (%)
$P_{(0,(0,0))}$	0.09659475	0.09248844	–4.44
$P_{(0,(0,1))}$	0.10355416	0.11048619	+6.27
$P_{(0,(1,0))}$	0.19054385	0.18997625	–0.30
$P_{(0,(1,1))}$	0.26508439	0.26171729	–1.29
$P_{(0,(2,0))}$	0.08605149	0.08351455	–2.91
$P_{(0,(2,1))}$	0.12645847	0.12860892	+1.67
$P_{(1,(0,0))}$	0.00000004	0.00000000	N/A
$P_{(1,(0,1))}$	0.00000002	0.00000000	N/A
$P_{(1,(1,0))}$	0.00000002	0.00000000	N/A
$P_{(1,(1,1))}$	0.00000001	0.00000000	N/A
$P_{(1,(2,0))}$	0.00000000	0.00000000	N/A
$P_{(1,(2,1))}$	0.00000000	0.00000000	N/A
$P_{(2,(0,0))}$	0.01720326	0.01587302	–8.38
$P_{(2,(0,1))}$	0.06709292	0.07074116	+5.16
$P_{(2,(1,0))}$	0.00860163	0.00824897	–4.28
$P_{(2,(1,1))}$	0.03354646	0.03374578	+0.59
$P_{(2,(2,0))}$	0.00107521	0.00024997	–330.00
$P_{(2,(2,1))}$	0.00419331	0.00424947	+1.33
ε	0.042713551	0.043483995	+1.77

6.2 EXAMPLE #2

$B = 20$
$C = 27$
$M = 2$
$N_1 = 2\ \alpha_1 = 0.2\ \beta 1 = 0.5\ R_1 = 20$
$N_2 = 3\ \alpha_2 = 0.1\ \beta 2 = 0.8\ R_2 = 15$

We denote P_i as the steady-state probability that the buffer content of the multiplexer at the start of a time slot is i and ϵ as the steady-state cell-loss probability. Relative error is defined as

$$100 \times \frac{\textit{simulation result} - \textit{analysis result}}{\textit{simulation result}}.$$

Table 2. ATM multiplexing algorithm example #2.

	Analysis	Simulation	Relative Error (%)
P_0	0.42601402	0.425331493	–0.16
P_1	0.05139291	0.050258995	–2.26
P_2	0.00656487	0.006659867	+1.43
P_3	0.01255233	0.012419752	–1.07
P_4	0.00684444	0.006959861	+1.66
P_5	0.00162335	0.001459971	–11.19
P_6	0.01994044	0.021119578	+5.58
P_7	0.00242924	0.002219956	–9.43
P_8	0.11457789	0.113537729	–0.92
P_9	0.01794887	0.017779644	–0.95
P_{10}	0.00251679	0.002519950	+0.13
P_{11}	0.00524285	0.005039899	–4.03
P_{12}	0.00245564	0.002339953	–4.94
P_{13}	0.09279831	0.093138137	+0.36
P_{14}	0.00826216	0.008079838	–2.26
P_{15}	0.00100513	0.000919982	–9.26
P_{16}	0.02671924	0.027779444	+3.82
P_{17}	0.00450124	0.004579908	+1.72
P_{18}	0.00161537	0.001699966	+4.98
P_{19}	0.00276439	0.002599948	–6.32
P_{20}	0.19223051	0.193556129	+0.68
ε	0.080323504	0.082366904	+2.48

7. CONCLUSION

Both simulation studies affirm the validity of this analysis. The main drawback of the procedure is that both its time and space complexities are exponential in M; therefore, it is impractical for evaluating multiplexers with interesting values of M, N_i, and B. To circumvent this difficulty, we propose an approximation multiplexer model to evaluate scenarios in which N_i's and M are large.

8. AN ALTERNATE MULTIPLEXER MODEL

In the previous section, we analyzed an ATM statistical multiplexer subject to heterogeneous groups of on-off traffic sources. The durations of the on and off periods of a traffic source are geometrically distributed with parameters α and β, respectively. A sample path of the state of a traffic source when α and β are close to 1 is illustrated in figure 3. Notice that the burst periods tend to last a long time; that is, given that the state-of-the-traffic source is on at time k, then it is highly probable that the traffic source is still on at time $k+\delta k$ even if δk is large. When cell generation tends to cluster together, we say that the generation process is positively correlated (figure 3). The correlativeness of a traffic source has a big impact on the cell-loss behavior at a multiplexer, particularly when the

buffer size of the multiplexer is large, since the large buffer permits the cells to interact. Since the on-off traffic source model proposed in the previous section can account for the correlative nature of cell arrivals, we refer to that on-off model as the *correlated burst on-off model.* Under certain asymptotic conditions, the correlated burst on-off model may be justifiably replaced by a simpler non-correlated burst on-off model, which is described in the following section.

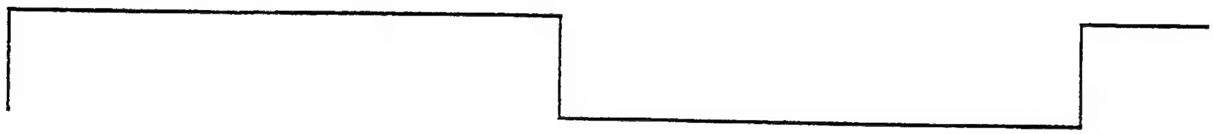

Figure 3. Positively correlated traffic generation.

Consider the sample paths of the states of two traffic sources, as shown in figure 4. Sample path #1 is constructed according to the correlated burst on-off model with $\alpha = 0.9$ and $\beta = 0.9$. Note that

$$\frac{\frac{1}{1-\alpha}}{\frac{1}{1-\alpha}+\frac{1}{1-\beta}} = 0.5.$$

Sample path #2 (figure 4) is constructed such that, at each time slot, the traffic source is on with probability 0.5 and off with probability 1 – 0.5; that is, the traffic source turns on and off at some time, k, with the same probability, independent of the k. We refer this on-off traffic model as the *random burst on-off* model. Notice that over a long observation period, the total durations of all the on periods of both sample paths are approximately the same; however, path #1 (figure 4) is much more positively correlated, and therefore, would effect a much higher cell loss at a multiplexer.

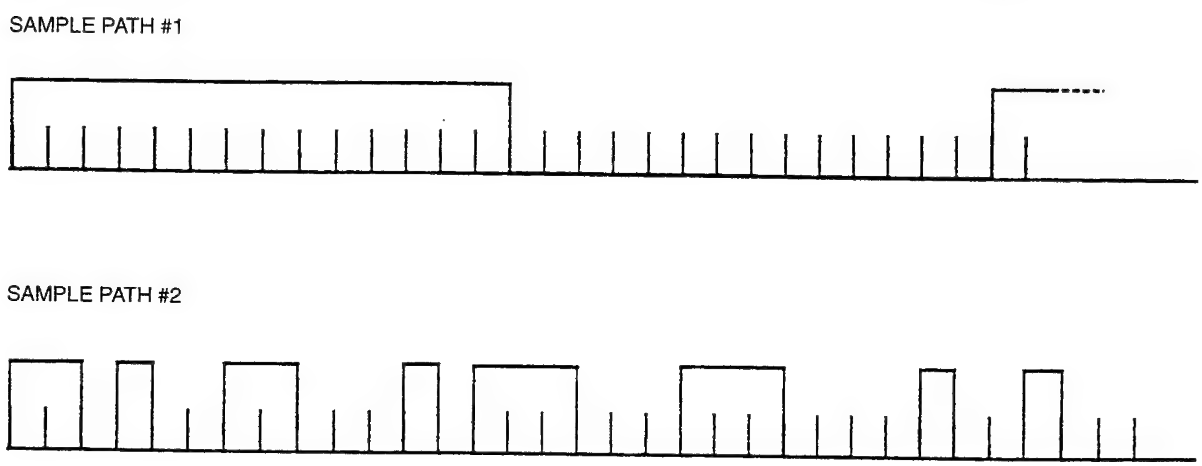

Figure 4. Sample paths of two types of on-off traffic sources.

Now consider the superpositions of three sample paths of the correlated burst on-off type and of three sample paths of the random burst type, as shown in figures 5 and 6. The two superpositions appear to be similar to each other, even though the sample paths that constituted them are quite different. The superposition of N sample paths of the correlated burst on-off model is stochastically similar to the superposition of sample paths of the random burst on-off model, when N is large. The correlativeness of an individual traffic stream is smoothed by the superposition. Extensive simulation studies have confirmed this interesting phenomenon.

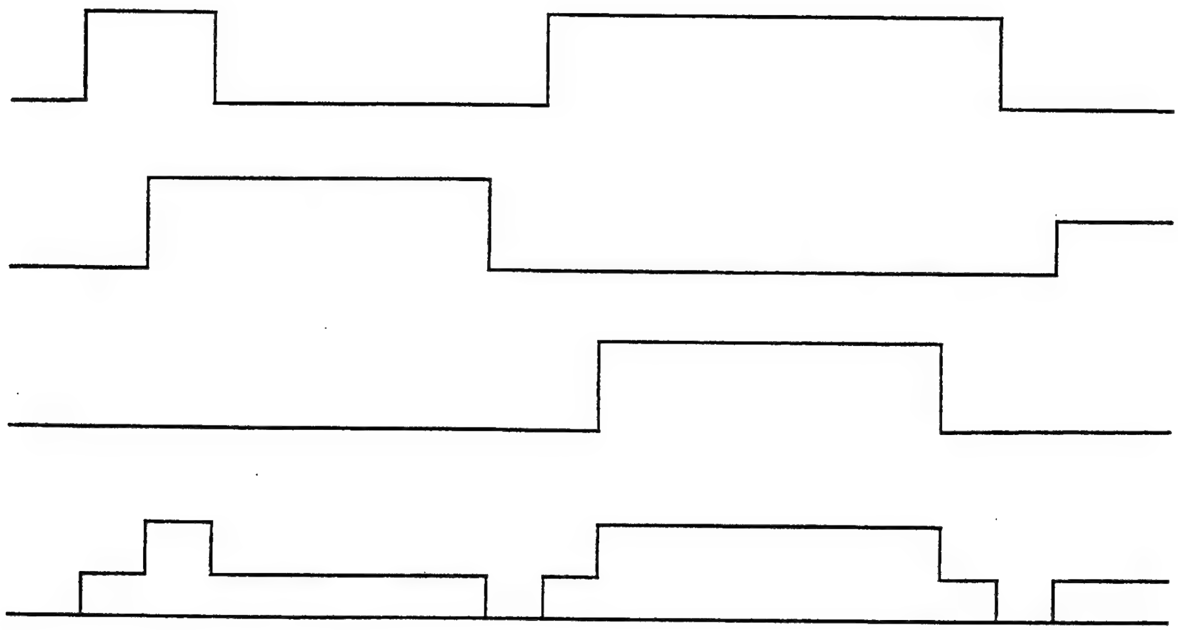

Figure 5. Superposition of correlated on-off sources.

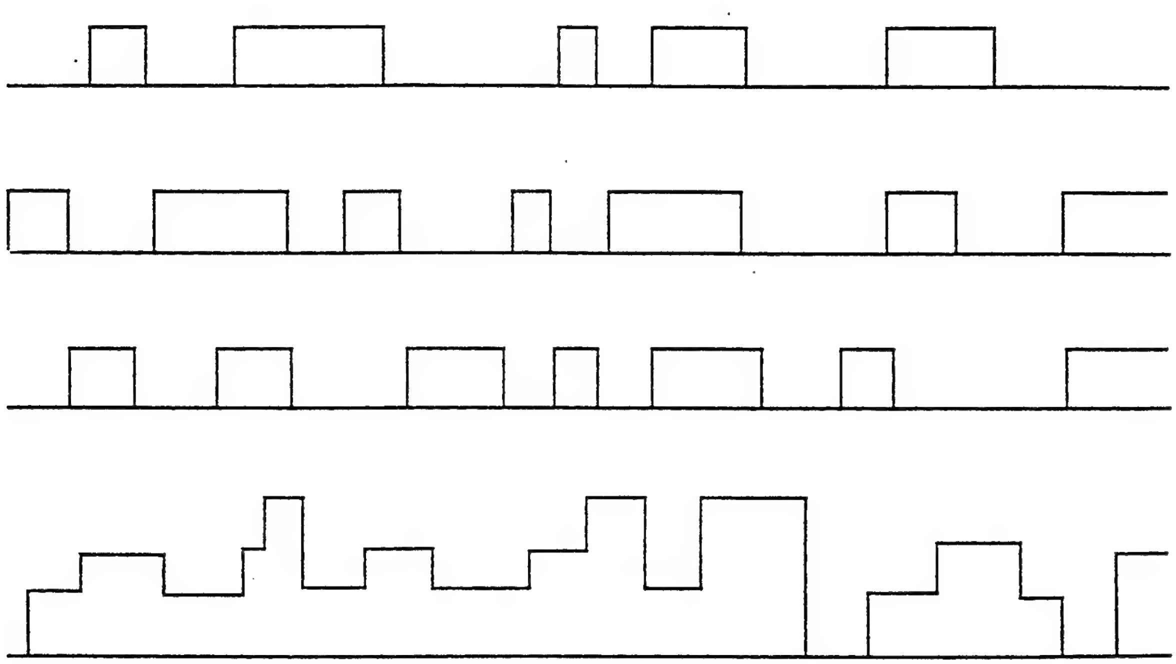

Figure 6. Superposition of random on-off sources.

The random burst on-off traffic streams have simpler analytical property and, if they were used as the traffic streams for a multiplexer, would simplify the analysis considerably; therefore, we propose to use random burst on-off traffic sources whenever the number of traffic sources of a correlated burst on-off traffic type is large. The modified multiplexer model with *random burst* on-off model is as follows:

- M is the number of different traffic types
- N_i is the number of traffic sources of the i-th type i=1, ..., M
- R_i is the number of cells generated by an i-th type traffic source in a time slot during an on period
- B is the maximum number of cells that may be buffered at the multiplexer
- C is the number of cells that may be transmitted by the output link of the multiplexer per time slot; Assume that $1 \leq C$

We make the following assumptions:

a. An *on* period of a traffic source in which the i-th type lasts X_i time slots, where X_i is *geometrically* distributed with parameters α_i

b. An *off* period of a traffic source in which the i-th type lasts Y_i time slots, where Y_i is *geometrically* distributed with parameter $1 - \alpha_i$

c. The number of cells at the buffer at a time k+1 is computed as $\max(\min(i + \underline{R} \cdot \underline{J} - C, B), 0)$, where i is the number of cells in the buffer at time k; $\underline{J}$ is the state of the traffic sources at the start of time k

d. $\sum_{i=1}^{M} R_i N_i \alpha_i < C$

e. $\sum_{i=1}^{M} R_i N_i > C$

Notice that the only difference between the correlated burst on-off model and the random burst model is that, in the correlated model, the on and off periods are determined by two parameters, α_i and βi, whereas in the random burst model, the periods are determined by only one parameter, α_i.

We define the following variables:

- B_k the number of cells in the buffer at the beginning of the k-th time slot
- J_{ik} the number of the i-th type traffic sources that are on at the beginning of k-th time slot
- $S(M, \underline{N}) = \{(j_1, j_2, \ldots, j_M) \mid 0 \leq j_i \leq N_i\, i = 1, 2, \ldots, M\}$
- $\underline{J}_k = (J_{1k}, J_{2k}, \ldots, J_{Mk}) \in S(M, \underline{N})$
- $Pr(B_{k+1} = j \mid B_k = i, \underline{J}_k = \underline{j})$ the probability that $B_{k+1} = j$, given that $B_k = i$ and $\underline{J}_k = \underline{j}$
- $Pr(B_{k+1} = j \mid B_k = i)$ the probability that $B_{k+1} = j$, given that $B_k = i$
- $Pr(B_{k+1} = j)$ the probability that $B_{k+1} = j$

We find that

$$Pr(B_{k+1} = j \mid B_k = i, \underline{J}_k = \underline{j}) = \xi\{j = \max(\min(i + \underline{R} \cdot \underline{j} - C, B), 0)\}, \tag{11}$$

$$Pr(B_{k+1} = j \mid B_k = i) = \sum_{\underline{j} \in S(M, \underline{N})} \xi\{j = \max(\min(i + \underline{R} \cdot \underline{j} - C, B), 0)\} \Pr(\underline{J}_k = \underline{j}), \tag{12}$$

$$Pr(B_{k+1} = j) = \sum_{i=0}^{B}\left(\sum_{\underline{j} \in S(M,\underline{N})} \xi\{j = \max(\min(i + \underline{R} \cdot \underline{j} - C, B), 0)\} Pr(\underline{J}_k = \underline{j})\right) Pr(B_k = i) . \tag{13}$$

In steady state, the number of cells in the buffer, L, is independent of the time, k; $\underline{J}$, the number of on traffic sources of each type at a time slot, is also independent of the time, k; therefore, we have the following fixed-point relation for $Pr(L = i)$ for i=0, 1, ..., B:

$$Pr(L = j) = \sum_{i=0}^{B}\left(\sum_{\underline{j} \in S(M,\underline{N})} \xi\{j = \max(\min(i + \underline{R} \cdot \underline{j} - C, B), 0)\} Pr(\underline{J} = \underline{j})\right) \Pr(L = i) , \tag{14}$$

where

$$Pr(\underline{J} = \underline{j}) = \prod_{i=1}^{M}\binom{N_i}{j_i}\alpha_i^{j_i}(1-\alpha_i)^{N_i - j_i} . \tag{15}$$

The $Pr(L = i)$'s may be obtained iteratively. We formulated the following iteration procedure to obtain the distribution function of the variable, L. P_i is used to denote $Pr(L = i)$.

$k = 0;$

$(p^k)_i = \frac{1}{B}$ for $i=1,2,..,B$

$(p^{k+1})_i = 0$ for $i=1,2,..,B$

While ($\left|p^{k+1}{}_i - p^k{}_i\right| \geq \delta$ for $0 \leq i \leq B$)

{

$k = k+1$;

For $0 \leq j \leq B$

{

pick any ε_k such that $0 < \varepsilon_k < 1$;

$$p^k{}_j = (1 - \varepsilon_k)p^{k-1}{}_j + \varepsilon_k \sum_{i=0}^{B}\left[\sum_{\underline{j} \in S(M,\underline{N})} \xi\left\{j = max(\min(i + \underline{R} \cdot \underline{j} - C, B), 0)\right\} Pr(\underline{J} = \underline{j})\right] p^{k-1}{}_i$$

};

$$S = \sum_{j=0}^{B} p^k{}_j ;$$

For $0 \le i \le B$

{

$$p^k{}_i = \frac{p^k{}_i}{S} ;$$

};

};

The procedure terminates whenever it has found a consistent set of $p^k_j = Pr(L = j)$'s.

We conclude that there exists a unique solution to the iteration scheme, since the system is ergodic. Note that the existence of a solution does not guarantee convergence of the iteration. Extensive computational experience suggests that the procedure always converges. Each iteration step requires $O(B^2 \prod_{i=1}^{M} N_i)$ computations; therefore, the scheme is not practical for interesting values of N_i and M. We will propose a heuristic that would considerably reduce the computational complexity of the procedure.

Once that $Pr(L = i)$ for i=0, 1, 2, ..., B are obtained, we may derive the *steady-state cell-loss probability* of the multiplexer as follows:

We define the variables:

- $NL(k,\underline{j})$ the number of cells that are lost in a time slot, given that there are k cells buffered at the beginning of the slot and that $\underline{j}$ is the state of the traffic sources at the beginning of the time slot
- $NL(k)$ the number of cells that are lost in a time slot, given that there are k cells buffered at the beginning of the slot
- $NA(k,\underline{j})$ the number of cells that arrive in a time slot, given that there are k cells in the beginning of the slot and $\underline{j}$ is the state of the traffic sources at the beginning of the time slot
- $NA(k)$ the expected number of cells that arrive in a time slot, given that there are k cells in the beginning of the slot and $\underline{j}$ is the state of the traffic sources at the beginning of the time slot
- NA the expected number of cells that arrive in a time slot
- NL the expected number of cell losses in a time slot
- C_L the steady-state cell-loss probability of the multiplexer

We find that

$$NL(k,\underline{j}) = \begin{cases} \underline{R} \cdot \underline{j} - (B - k + C) & B - k + C \le \underline{R} \cdot \underline{j} \\ 0 & \text{otherwise} \end{cases} \tag{16}$$

$$NL(k) = \sum_{\underline{j} \in S(M,\underline{N})} NL(k,\underline{j}) \Pr(\underline{J} = \underline{j}) , \tag{17}$$

$$NL = \sum_{k=0}^{B} \{NL(k)\Pr(L = k)\}\,, \tag{18}$$

$$NA(k,\underline{j}) = \underline{R} \cdot \underline{j} = \sum_{i=1}^{M} R_i\, j_i\,, \tag{19}$$

$$NA(k) = \sum_{\underline{j} \in S(M,\underline{N})} NA(k,\underline{j}) Pr(\underline{J} = \underline{j}) = \sum_{\underline{j} \in S(M,\underline{N})} \left\{(\underline{R} \cdot \underline{j}) Pr(\underline{J} = \underline{j})\right\}, \tag{20}$$

$$NA = \sum_{\underline{j} \in S(M,\underline{N})} \left\{(\underline{R} \cdot \underline{j}) \Pr(\underline{J} = \underline{j})\right\}. \tag{21}$$

The cell-loss probability during a time slot is

$$C_L = \frac{NL}{NA} = \frac{\sum_{k=0}^{B} \left\{ \sum_{\underline{j} \in S(M,\underline{N})} (\underline{R} \cdot \underline{j} - (B - k + C)) Pr(\underline{J} = \underline{j}) \xi(\underline{R} \cdot \underline{j} \ge B - k + C) \right\} Pr(L = k)}{\sum_{\underline{j} \in S(M,\underline{N})} \left\{(\underline{R} \cdot \underline{j}) Pr(\underline{J} = \underline{j})\right\}}. \tag{22}$$

From the above expression, the cell-loss probability may be obtained in principle; however, the computational complexity is exponential to the sizes of the parameters N_i's, R_i's, and M. We propose a heuristic that will make the requisite computation feasible.

Notice that

$$\sum_{i=0}^{B} \left(\sum_{\underline{j} \in S(M,\underline{N})} \xi\left\{j = \max(\min(i + \underline{R} \cdot \underline{j} - C, B), 0)\right\} \Pr(\underline{J} = \underline{j}) \right) = \sum_{i=0}^{B} \sum_{\underline{j} \in S_{i,j,\underline{j}}} Pr(\underline{J} = \underline{j})\,, \tag{23}$$

where

$$S_{i,j,\underline{j}} = \left\{\underline{j} \mid j = \max(\min(i + \underline{R} \cdot \underline{j} - C, B), 0)\right\}.$$

For a fixed pair (i, j), the set, $S_{i,j,\underline{j}}$, should be much smaller than $S(M,\underline{N})$. We develop a procedure that enumerates the set $S_{i,j,\underline{j}}$ without testing each $\underline{j} \in S(M,\underline{N})$.

We partition $S_{i,j,\underline{j}}$ into $S_{i,j,\underline{j}}[j = 0], S_{i,j,\underline{j}}[1 \le j < B]$ and $S_{i,j,\underline{j}}[B \le j]$, where

$$S_{i,j,\underline{j}}[j = 0] = \left\{\underline{j} \in S(M,\underline{N}) : \underline{R} \cdot \underline{j} \le C{-}i\right\}, \tag{24}$$

$$S_{i,j,\underline{j}}[1 \le j < B] = \left\{\underline{j} \in S(M,\underline{N}) : \underline{R} \cdot \underline{j} = C + j{-}i\right\}, \tag{25}$$

$$S_{i,j,\underline{j}}[B = j] = \left\{\underline{j} \in S(M,\underline{N}) : \underline{R} \cdot \underline{j} \ge C + B{-}i\right\}. \tag{26}$$

We define the following sets:

- $S_{[1..M]}[\le k] = \left\{\underline{j} \in S(M,\underline{N}) \;\middle|\; \sum_{i=1}^{M} R_i j_i \le k\right\},$

- $S_{[1..M]}[= k] = \left\{\underline{j} \in S(M,\underline{N}) \;\middle|\; \sum_{i=1}^{M} R_i j_i = k\right\},$

- $S_{[1..M]}[\ge k] = \left\{\underline{j} \in S(M,\underline{N}) \;\middle|\; \sum_{i=1}^{M} R_i j_i \ge k\right\},$

We formulate a procedure that will enumerate the sets $S_{[1...M]}[\leq k], S_{[1...M]}[= k]$ and $S_{[1...M]}[\geq k]$ without testing each $\underline{j} \in S(M, \underline{N})$.

The notations are as follows:

- $S_i[= k] = \{j \mid 0 \leq j, \leq N_i, Rj = k\}$,
- $A \times B = \{(i,j) \mid i \in A, j \in B\}$,
- $S_{[1...m]}[= k] = \begin{cases} \sum_{j \in S_i[=k]} (S_{[1...m-1]}[= (k - jR_m)] \times \{j\}) & M \geq m > 1 \\ \{j \mid 0 \leq j \leq N_1,\ jR_1 = k\} & m = 1 \end{cases}$

The summation symbol $\sum$ represents the *union* set operation.

We define

- $S_i[\leq k] = \{j \mid 0 \leq j \leq N_i,\ Rj \leq k\}$
- $S_{[1...m]}[\leq k] = \begin{cases} \sum_{j \in S_i[\leq k]} (S_{[1...m-1]}[\leq (k - jR_m)] \times \{j\}) & M \geq m > 1 \\ \{j \mid 0 \leq j \leq N_1,\ jR_1 \leq k\} & m = 1 \end{cases}$

We introduce the notation

- $S_i = \{j \mid 0 \leq j \leq N_i\}$,
- $S_{[1...m]}[\geq k] = \begin{cases} \sum_{j \in S_m} (S_{[1...m-1]}[\geq (k - jR_m)] \times \{j\}) & M \geq m > 1 \\ \{j \mid 0 \leq j \leq N_1,\ jR_1 \geq k\} & m = 1 \end{cases}$

Using this enumeration scheme, the summation $\sum_{\underline{j} \in S(M, \underline{N})}$ of the iterative procedure may be executed without testing each $\underline{j} \in S(M, \underline{N})$. This reduces computation considerably. In the worst case, our computational procedure to calculate the exact cell-loss probability is still $O(B^2 \prod_{i=1}^{M} N_i)$, although its average complexity is much lower. For the case that N_i's are large, we develop the following efficient procedure to *approximate* the cell-loss probability of the multiplexer.

9. DIFFUSION APPROXIMATION

When N_i is large for each i=1, 2, ..., M, *diffusion* approximation may be used to estimate $Pr(\underline{J} = \underline{j})$. If N_i is large, $N_i\alpha_i \gg 1$ and $N_i\alpha_i(1 - \alpha_i) \gg 1$, then by the *Laplace-DeMoivre Theorem*,

$$Pr(J_i = j) \sim \frac{1}{\sqrt{2\pi\sigma_i^2}} e^{\left\{\frac{-(j-\mu_i)^2}{2\sigma_i^2}\right\}}, \tag{27}$$

where $\mu_i = N_i\alpha_i$ and $\sigma_i^2 = N_i\alpha_i(1 - \alpha_i)$.

We approximate the discrete random variable, J_i, with a continuous variable, Y_i. The probability distribution for Y_i is

$$Pr(Y_i \le x) = \frac{1}{K_i\sqrt{2\pi\sigma_i^2}} \int_0^x e^{\left\{\frac{-(\tau-\mu_i)^2}{2\sigma_i^2}\right\}} d\tau \qquad 0 < x \le N_i, \tag{28}$$

where $K_i = \frac{1}{\sqrt{2\pi\sigma_i^2}} \int_0^{N_i} e^{\left\{\frac{-(\tau-\mu_i)^2}{2\sigma_i^2}\right\}} d\tau$.

Y_i is approximately *Gaussian.*

Finally, if we define

$$Y \equiv \sum_{i=1}^{M} R_i Y_i, \tag{29}$$

Y is also approximately *Gaussian* with a mean of $\mu = \sum_{i=1}^{M} R_i N_i \alpha_i$ and a variance $\sigma^2 = \sum_{i=1}^{M} R_i^2 N_i \alpha_i (1 - \alpha_i)$.

The distribution function of Y is

$$Pr(Y \le x) = \frac{1}{K\sqrt{2\pi\sigma^2}} \int_0^x e^{\left\{\frac{-(\tau-\mu)^2}{2\sigma^2}\right\}} d\tau \qquad 0 < x \le \sum_{i=1}^{M} R_i N_i, \tag{30}$$

where $K = \frac{1}{\sqrt{2\pi\sigma^2}} \int_0^{\sum_{i=1}^{M} R_i N_i} e^{\left\{\frac{-(\tau-\mu)^2}{2\sigma^2}\right\}} d\tau$.

We conclude that

$$\sum_{\underline{j} \in S_{i,j,\underline{j}}} \Pr(\underline{J} = \underline{j}) \sim \xi\{j = 0\} Pr(Y \le C - i) \tag{31}$$

$$+ \xi\{0 < j < B\} Pr(C + j - i - 1 \le Y \le C + j - i) + \xi\{B = j\}(1 - Pr(Y \le C + B - i)).$$

The Pr(L = i)'s may be obtained from the following iteration scheme in which $\sum_{\underline{j} \in S_{i,j,\underline{j}}} P(\underline{J} = \underline{j})$ is approximated by the expression of equation (31).

$k = 0;$

$(p^k)_i = \frac{1}{B}$ for $i=1,2,..,B$

$(p^{k+1})_i = 0$ for $i=1,2,..,B$

While $(\left| p^{k+1}{}_i - p^k{}_i \right| \geq \delta$ for $0 \leq i \leq B)$

{

$k = k+1$;

For $0 \leq j \leq B$

{

pick any ε_k such that $0 < \varepsilon_k < 1$;

$p^k{}_j = (1 - \varepsilon_k)p^{k-1}{}_j + \varepsilon_k \sum_{\underline{i} \in S_{i\,j\,\underline{j}}} Pr(\underline{J} = \underline{j})p^{k-1}{}_i$

};

$S = \sum_{j=0}^{B} p^k{}_j;$

For $0 \leq i \leq B$

{

$p^k{}_i = \frac{p^k{}_i}{S}$;

};

};

We also may approximate NL and NA as follows:

We note that

$$\sum_{\underline{j} \in S(M,\underline{N})} \left\{(\underline{R} \cdot \underline{j} - (B - k + C))Pr(\underline{J} = \underline{j})\right\} \equiv \sum_{\underline{j} \in S_{k,\underline{j}}} \left\{(\underline{R} \cdot \underline{j} - (B - k + C))\Pr(\underline{J} = \underline{j})\right\}, \qquad (32)$$

where $S_{k\underline{j}} \equiv \left\{\underline{j} \in S(M,\underline{N}) : B - k + C \leq \underline{R} \cdot \underline{j}\right\}$.

Again, we may approximate the sum

$$\sum_{\underline{j} \in S_{k,\underline{j}}} \left\{ (\underline{R} \cdot \underline{j} - (B-k+C)) \Pr(\underline{J} = \underline{j}) \right\} \sim \int_{a_k}^{\sum_{i=1}^{M} R_i N_i} (\tau - a_k) f_Y(\tau) d\tau \,, \tag{33}$$

where $a_k = B - k + C$ and $f_Y(\tau) = \frac{1}{K\sqrt{2\pi\sigma^2}} e^{\left\{ \frac{-(\tau-\mu)^2}{2\sigma^2} \right\}}$.

Finally,

$$NA \sim \sum_{i=1}^{M} \{R_i N_i \alpha_i\} \,. \tag{34}$$

The cell-loss probability is

$$C_L \sim \frac{\sum_{k=0}^{B} \left(\int_{a_k}^{\sum_{i=1}^{M} R_i N_i} (\tau - a_k) f_Y(\tau) d\tau \right) \Pr(L = k)}{\sum_{i=1}^{M} \{R_i N_i \alpha_i\}} \tag{35}$$

Using this approximation, the computation at each iteration step is reduced to the evaluation of B^2 error function integrals. The complexity of evaluating an integral is $O(\sum_{i=1}^{M} R_i N_i)$; therefore, the overall complexity of the approximation procedure is $O(B^2 \sum_{i=1}^{M} R_i N_i)$. If one evaluates each integral using a fixed number, rather than $O(\sum_{i=1}^{M} R_i N_i)$, of steps, the complexity of the algorithm is reduced to $O(B^2)$. We performed several experiments using the $O(B^2)$ algorithm and found that it yielded virtually the same results as the $O(B^2 \sum_{i=1}^{M} R_i N_i)$ algorithm.

10. NUMERICAL EXAMPLES II

We conducted several simulation studies to evaluate the accuracy of exact and approximation algorithms that estimate the cell-loss probability of the random-burst multiplexer model. The studies showed good agreement. Tables 3 and 4 list the results from two of the simulation studies.

10.1 EXAMPLE #1 (EXACT ANALYSIS)

$B = 10$
$C = 50$
$M = 2$

$N_1 = 3\ \alpha_1 = 0.2\ R_1 = 40$
$N_2 = 10\ \alpha_2 = 0.8\ R_2 = 1$

We denote P_i as the steady-state probability that the buffer content of the multiplexer at the start of a time slot is i and ϵ as the steady-state cell-loss probability. Relative error is defined as $100 \times \frac{simulation - analysis}{simulation}$.

Table 3. Exact algorithm.

	Analysis	Simulation	Relative Error (%)
P_0	0.82978	0.83290	–0.376
P_1	0.00106	0.00110	–3.78
P_2	0.00148	0.00150	–1.35
P_3	0.00228	0.00220	+3.51
P_4	0.00350	0.00330	+5.71
P_5	0.00524	0.00500	+4.58
P_6	0.00816	0.00780	+4.41
P_7	0.01230	0.01170	+4.88
P_8	0.01444	0.01440	+0.28
P_9	0.01150	0.01770	–1.74
P_{10}	0.110260	0.10850	+1.60
ε	0.1069174	0.105817096	+1.03

10.2 EXAMPLE #2 (APPROXIMATION ANALYSIS)

$B = 20$
$C = 3556$
$M = 5$
$N_1 = 50\ R_1 = 10\ \alpha_1 = 0.6$
$N_2 = 50\ R_2 = 30\ \alpha_2 = 0.7$
$N_3 = 50\ R_3 = 20\ \alpha_3 = 0.5$
$N_4 = 50\ R_4 = 50\ \alpha_4 = 0.3$
$N_5 = 50\ R_5 = 60\ \alpha_5 = 0.2$

Table 4. Approximation algorithm.

	Analysis	Simulation	Relative Error (%)
P_0	0.9096	0.9079	+0.19
P_4	0.0050	0.0060	–20.00
P_{14}	0.0060	0.0057	+5.00
P_{20}	0.0794	0.0802	–1.01
ε	0.003015543	0.002997045	+0.61

11. APPLICATION: ATM EFFICIENCY

How efficient is ATM? This is one of the most frequently asked questions by ATM network designers. The following example illustrates how the algorithms formulated in the preceding sections would aid in addressing this question.

To simplify our discussion, we consider an ATM network consisting of only one transmission link. The network is to support many connections, each carrying the same traffic type. We wish to determine what is the maximum link efficiency achievable such that the quality of service requirement (QOS) of each connection supported is satisfied. The answer depends on the following parameters:

- C the capacity of the transmission link
- B the size of the buffer
- N the number of connections being supported
- $\underline{TP}$ the traffic profile of the connections
- $\underline{Q}$ the QOS requirement of the connections

We make the following simplifying assumptions:

- The *traffic profile,* $\underline{TP}$, is specified in terms of the *peak traffic rate*, the *mean traffic rate*, and the *expected burst period* of a connection.
- The QOS requirement, $\underline{Q}$, is usually defined in terms of both the maximum cell-loss rate and the maximum end-to-end delay tolerable by the connection. To simplify the discussion, we assume that the QOS requirement is measured only in terms of the maximum *cell-loss* rate tolerable by a connection .

The transmission link is *statistically shared (multiplexed)* among N connections. When the total cell-generation rate of the N connections is below the capacity of the link, C, no loss occurs; however, when the instantaneous total traffic rate exceeds C, the excess cells will be stored in the buffer. When the buffer is full, cells will be dropped.

We will use the following examples to illustrate how the parameters influence the answer to our question.

For each of the two examples below, we assume the following:

The network is to support packetized voice connections. We assume that the voice encoder of a connection would sample the incoming voice signal at 8000 samples/sec and that each sample is encoded into an 8-bit unit. A cell is packed with 48 samples (bytes). We also assume that the encoder has a silence detector and, therefore, would not generate samples during the period that the talker is silent. We assume that the *expected burst* period, the interval during which the talker speaks, is 0.96 seconds; the *expected silence* period, the interval during which the talker is silent, is 1.60 seconds.

Based on the above assumptions, the traffic profile of each voice connection is as follows:

- The peak cell rate of the connection is 167 cells/sec.
- The mean cell rate of the connection is 62 cells/sec.
- The expected burst period, the period during which the voice source would generate traffic at its peak rate, is 0.96 seconds.

- It is assumed that the cell-loss requirement of each connection is 10^{-7} and that an overall cell-loss rate of 10^{-8} at the transmission link suffices to effect a cell-loss rate of 10^{-7} to each connection.

11.1 EXAMPLE 1

We assume that the link can transmit up to 10 Mbits/sec (~26000 cells per second).

The buffer may store up to 100 cells.

What is the maximum efficiency achievable by the link? That is, what is the maximum number of voice connections that can be supported by the link such that the cell-loss requirement of each connection is satisfied?

We use the multiplexer model with correlated burst on-off traffic sources. According to the model, the maximum number of connections that can be supported by the link is $N = 310$. The efficiency is $\frac{310 \times 62}{26000} = 0.74$.

11.2 EXAMPLE 2

We retain the assumptions made in example 1, with the *exception that the link capacity is only 1 Mbits/sec.*

The answer to the same question is $N = 20$. The efficiency is $\sim \frac{20 \times 62}{2600} = 0.48$.

Notice that the network in example 1 has a much higher *statistical gain* than that of example 2. This reenforces the proven principle that networks with higher capacity transmission links can achieve significantly higher statistical gain than low-capacity networks.

12. FUTURE WORK AND SUMMARY

In this report, a discrete-time ATM statistical multiplexer with correlated burst on-off traffic sources is proposed. Algorithms to compute both the exact steady state and transient cell-loss probabilities of the multiplexer have been formulated. Simulation studies confirm the validity of the exact algorithms. Unfortunately, the computational requirement of the algorithms prohibits their use in evaluating all ATM multiplexing scenarios. As a remedy, we proposed a multiplexer model with random burst on-off traffic sources. This model has similar cell-loss behaviors as the correlated burst model's when the number of traffic sources of each traffic type being multiplexed is large. For the random burst model, we formulated an algorithm to compute the multiplexer's cell-loss rate (transient and steady state). The time complexity of the exact algorithm is still exponential; for this reason, we developed an efficient approximation algorithm to compute the cell-loss rate. Extensive simulation studies had been performed to evaluate the accuracy of the approximation algorithm, and for all parameters of practical interest, the algorithmic results and the simulation outputs showed perfect agreement. Finally, we illustrated the practical use of the algorithms presented in this report to determine the expected efficiency of an idealized ATM network.

The multiplexer model with correlated burst on-off traffic sources is considered to be one of the most general multiplexer models that has ever been proposed. As far as we are able to determine, no exact solutions (closed-form or algorithmic) to comparably general models are known. The contribution of this paper is the development of exact, easily implementable, and numerically stable

algorithms to compute the transient and steady-state cell-loss probabilities of a faithful ATM statistical multiplexer model. The main drawback of the solution is that it is *algorithmic* rather than in closed-form. An obvious future work on this subject is to find a closed-form solution for the cell-loss probability of this model.

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REPORT DOCUMENTATION PAGE

Form Approved
OMB No. 0704-0188

1. AGENCY USE ONLY *(Leave blank)*	2. REPORT DATE	3. REPORT TYPE AND DATES COVERED
	September 1995	Final: September 1995

4. TITLE AND SUBTITLE

DISCRETE-TIME ANALYSIS OF AN ATM MULTIPLEXER

5. FUNDING NUMBERS

PE: 0601152N
AN: DN304006

6. AUTHOR(S)

A. Shum

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)

Naval Command, Control and Ocean Surveillance Center (NCCOSC)
RDT&E Division
San Diego, CA 92152–5001

8. PERFORMING ORGANIZATION REPORT NUMBER

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)

Office of Naval Research
800 N. Quincy Street
Arlington, VA 22217–5000

10. SPONSORING/MONITORING AGENCY REPORT NUMBER

TR 1704

11. SUPPLEMENTARY NOTES

12a. DISTRIBUTION/AVAILABILITY STATEMENT

Approved for public release; distribution is unlimited.

12b. DISTRIBUTION CODE

13. ABSTRACT *(Maximum 200 words)*

This report proposes a discrete-time model of an Asynchronous Transfer Mode (ATM) statistical multiplexer subject to heterogeneous groups of ON-OFF traffic sources with correlated burst periods. NRaD researchers developed algorithms to compute both the exact transient and steady-state cell-loss probabilities of the multiplexers. The results yielded by the algorithms have been compared to simulation outputs and they showed perfect agreement (within noise level). The algorithms will be invaluable for resolving basic ATM design issues such as buffer dimensioning, maximal loading, and admission control.

14. SUBJECT TERMS

Asynchronous Transfer Mode (ATM)
statistical multiplexer
steady state
cell-loss probability

15. NUMBER OF PAGES

34

16. PRICE CODE

17. SECURITY CLASSIFICATION OF REPORT	18. SECURITY CLASSIFICATION OF THIS PAGE	19. SECURITY CLASSIFICATION OF ABSTRACT	20. LIMITATION OF ABSTRACT
UNCLASSIFIED	UNCLASSIFIED	UNCLASSIFIED	SAME AS REPORT

21a. NAME OF RESPONSIBLE INDIVIDUAL	21b. TELEPHONE *(include Area Code)*	21c. OFFICE SYMBOL
A. Shum	(619) 553–4968	Code 827

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